Calculation of Gas Density and Viscosity

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COURSE CONTENT

1.   Introduction

The density and/or viscosity of a gas is often needed for some other calculation, such as pipe flow or heat exchanger calculations. This course contains discussion of, and example calculation of, the density and viscosity of a specified gas at a given temperature and pressure.

If the gas temperature is high relative to its critical temperature and the gas pressure is low relative to its critical pressure, then it can be treated as an ideal gas and its density can be calculated at a specified temperature and pressure using the ideal gas law.

If the density of a gas is needed at a temperature and pressure at which it cannot be treated as an ideal gas, however, then the compressibility factor of the gas must be calculated and used in calculating its density. In this course, the Redlich Kwong equation will be used for calculation of the compressibility factor of a gas.

The Sutherland formula can be used to calculate the viscosity of a gas at a specified temperature if the Sutherland constant is available for the gas. It will be discussed and used in example calculations. Another method for calculating the viscosity of air at a specified temperature and pressure will also be presented and discussed. Some of the equations that will be discussed and illustrated through examples are shown below.

$$\rho = P(MW)/RT \qquad \rho = MW*P/(Z*R*T)$$

$$Z^3 - Z^2 - qZ - r = 0 \qquad q = B^2 + B - A^2$$

$$A^2 = 0.42747 P_R/T_R^{2.5} \text{ and } B = 0.08664\, P_R/T_R$$

$$\mu = \mu_o \frac{T_o + C}{T + C} \left(\frac{T}{T_o}\right)^{3/2}$$

$$\text{air viscosity} = (1.2867/10^7)[\eta_o(T_r) + \Delta\eta(\rho_r)]$$

2.   Learning Objectives

Upon completion of this course, the student will

- Be able to calculate the density of a gas of known molecular weight at a specified temperature and pressure at which the gas can be treated as an ideal gas.

- Be able to calculate the compressibility factor for a gas at a specified temperature and pressure, using the Redlich-Kwong equation, if the molecular weight, critical temperature and critical pressure of the gas are known.

- Be able to calculate the density of a gas at a specified temperature and pressure for which the gas cannot be treated as an ideal gas, if the molecular weight, critical temperature and critical pressure of the gas are known.

- Be able to calculate the viscosity of a gas at a specified temperature if the Sutherland constant for the gas is known and the viscosity of the gas at a suitable reference temperature is known.

- Be able to calculate the viscosity of air at specified air temperature and pressure.

- Be able to make all of the calculations described in these learning objectives using either U.S. or S.I. units.

3.   Topics Covered in this Course

I.   Calculation of Ideal Gas Density

II.   Calculation of Real Gas Density

III.   Calculation of Gas Viscosity by Sutherland's Formula

IV.   Calculation of Air Viscosity at Specified Temperature and Pressure

4.   Calculation of Ideal Gas Density

The typical form for the Ideal Gas Law is: **PV = nRT**

The parameters in the equation with a consistent set of units are as shown below:

P is the absolute pressure of the gas in psia
V is the volume of the gas in ft^3
n is the number of slugmoles in the gas contained in volume, **V**
R is the ideal gas law constant, 345.23 psia-ft^3/slugmole-^{o}R
T is the absolute temperature of the gas in ^{o}R

The mass of the gas can be introduced into the equation by replacing **n** with **m/MW**, where **m** is the mass of the gas contained in volume **V** in slugs, and **MW** is the molecular weight of the gas (slugs/slugmole). The Ideal Gas Law then becomes:

$$\textbf{PV } = \textbf{ (m/MW)RT}$$

Solving the equation for m/V, which is the density of the gas gives:

$$\rho \textbf{ = m/V = P(MW)/RT}$$

With **P**, **R**, and **T** in the units given above, the gas density will be in slugs/ft^3. Note that 1 slug = 32.17 lbm, so if you want the gas density in lbm/ft^3, the value in slugs/ft^3 should be multiplied by 32.17.

S.I. Units: If working in S.I. units, the equations remain the same with the following units:

P is the absolute pressure of the gas in kPa
V is the volume of the gas in m^3
n is the number of kgmoles in the gas contained in volume, **V**

R is the ideal gas law constant, 8.3145 kN-m/kgmole-K
T is the absolute temperature of the gas in K

With these units for **P, V, R**, and T, the gas density will be in kg/m^3.

Critical Temperature and Pressure: As noted in the Introduction, in order to use the Ideal Gas Law to calculate a gas density, the gas temperature should be high relative to its critical temperature and the gas pressure should be low relative to its critical pressure. **Table 1** gives critical temperature, critical pressure and molecular weight for 16 gases in U.S. units. **Table 2** provides the same in S.I. units.

Table 1. Critical Temperature and Pressure and Molecular Weight -U.S.

Gas	Critical Temp, °F	Critical Press, psi	MW
Acetylene	95.23	890.2	26.04
Air	-220.9	547	28.96
Ammonia	270	1636	17.02
Argon	-188.17	710.4	39.95
Butane	305.53	550.6	58.12
Carbon Dioxide	87.8	1070	44.01
Carbon Monoxide	-220.5	508	28.011
Chlorine	291.23	1118	70.91
Ethane	89.93	706.6	30.07
Helium	-450.31	33	4.003
Hydrogen	-400	188.2	2.016
Methane	-116.4	673	16.044
Nitrogen	-232.6	493	28.0134
Oxygen	-181.5	731	31.9988
Propane	206.3	619	44.097
Sulfur Dioxide	315.63	1143.5	64.06

Table 2. Critical Temperature and Pressure and Molecular Weight - metric

Gas	Critical Temp, °C	Critical Press, atm	MW
Acetylene	35.17	60.57	26.04
Air	-140.5	37.25	28.96
Ammonia	132.2	111.29	17.02
Argon	-122.3	48.33	39.95
Butane	151.96	37.46	58.12
Carbon Dioxide	31	72.9	44.01
Carbon Monoxide	-140.3	34.53	28.011
Chlorine	144	76.05	28.0134
Ethane	32.18	48.07	31.9988
Helium	-267.85	2.24	44.097
Hydrogen	-240	12.8	2.016
Methane	-82.4	45.8	16.044
Nitrogen	-147	33.54	28.0134
Oxygen	-118.6	50.14	31.9988
Propane	96.9	42.1	44.097
Sulfur Dioxide	157.6	77.79	64.06

Example #1:

a) Calculate the density of air at -17.00 °F and 20.00 psig, assuming that the air can be treated as an ideal gas at those conditions.

b) Is it reasonable to assume ideal gas behavior for air at -17.00 °F and 20.00 psig?

Solution:

a) The absolute temperature and pressure need to be calculated as follows: T_{abs} = -17.00 + 459.67 °R = 442.67 °R and $P_{abs} = P_g + P_{atm}$ = 20.00 + 14.7 = 34.7 psia.

Substituting values into the ideal gas law (using 28.97 as the MW of air) gives:

$\rho = \textbf{MW*P/(R*T)} = 28.97*34.7/(345.23*442.67) = \textbf{0.00658 slugs/ft}^3$

If desired, the density can be converted to lbm/ft^3 by multiplying by the conversion factor, 32.17 lbm/slug.

$\rho = (0.000658 \text{ slugs/ft}^3)(32.17 \text{ lbm/slug}) = \textbf{0.2116 lbm/ft}^3$

b) The gas temperature (-17 ºF) is much greater than the critical temperature of air (-220.9 ºF) and the gas pressure (34.7 psia) is much less than the critical pressure of air (547 psia), so it would be reasonable to assume ideal gas behavior for air at this temperature and pressure.

Example #2:

a) Calculate the density of air at 10.00ºC and 100.0 kPa gauge pressure, assuming that it can be treated as an ideal gas at those conditions.

b) Is it reasonable to assume ideal gas behavior for air at 10 ºC and 100 kPa guage?

Solution:
a) The absolute temperature and pressure need to be calculated as follows: $\textbf{T}_{\textbf{abs}} = 10.00 + 273.15 \text{ K} = 283.15 \text{ K}$ and $\textbf{P}_{\textbf{abs}} = \textbf{P}_{\textbf{g}} + \textbf{P}_{\textbf{atm}} = 100.0 + 101.3 = 201.3$ kPa abs.

Substituting values into the ideal gas law (using 28.97 as the MW of air) gives:

$\rho = \textbf{MW*P/(R*T)} = 28.97*201.3/(8.3145*283.15) = \textbf{2.48 kg/m}^3$

b) The gas temperature (10 ºC) is much greater than the critical temperature of air (-140.5 ºC) and the gas pressure (201.3 kPa abs) is much less than the critical pressure of air (3773.4 kPa abs), so it would be reasonable to assume ideal gas behavior for air at this temperature and pressure. (Note that the critical temperature was converted from the 37.25 atm value in the table with the conversion factor, 101.3 kPa/atm.)
Spreadsheet Use for the Calculations: These calculations are rather straight-forward and not too difficult to do by hand, but they can be done

very conveniently with an Excel spreadsheet set up to calculate gas density with the Ideal Gas Law. **Figure 1** shows a screenshot of an Excel worksheet with the solution to **Example 1 (a)** and **Figure 2** shows a screenshot with the solution to **Example 2 (a).**
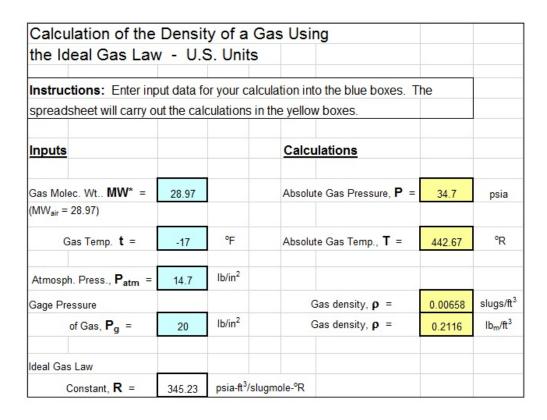

Figure 1. Screenshot of Solution to **Example #1 (a)**

Calculation of the Density of a Gas Using
the Ideal Gas Law - S.I. Units - (for air use MW - 29)

Instructions: Enter input data for your calculation into the blue boxes. The
spreadsheet will carry out the calculations in the yellow boxes.

Inputs			Calculations		
Gas Molec. Wt.. **MW*** =	28.97		Absolute Gas Pressure, **P** =	201.3	kPa
(MW$_{air}$ = 28.97)			(using P$_{atm}$ = 101.3 kN/m^2)		
Gas Temp. **t** =	10	°C	Absolute Gas Temp., **T** =	283.15	K
Atmosph. Press., **P$_{atm}$** =	101.3	kPa	Gas density, **ρ** =	2.48	kg/m^3
Gage Pressure					
of Gas, **P$_g$** =	100	kPa			
Ideal Gas Law					
Constant, **R** =	8.3145	kN-m/kgmole-K			

Figure 2. Screenshot of Solution to **Example #2 (a)**

5.   Calculation of Real Gas Density

In some cases, the Ideal Gas Law cannot be used to calculate the density of a gas because its temperature is too close to its critical temperature and/or its pressure is too close to its critical pressure. In that case, if the compressibility factor, **Z**, can be determined at the gas temperature and pressure, it can be used to calculate the gas density with the following equation:

$$\rho \;=\; MW*P/(Z*R*T)$$

The compressibility factor for a gas is, in general, a function of its reduced temperature (T_R) and reduced pressure (P_R), where reduced temperature is the absolute gas temperature divided by its absolute critical temperature and reduced pressure is the absolute gas pressure divided by its absolute critical pressure. Graphs, tables and equations are available for determining the compressibility factor at specified values for T_R and P_R. The Redlich-Kwong equation will be used in this book as a means of calculating **Z** as a function of T_R and P_R.

Calculation of the compressibility factor of a gas from the Redlich-Kwong equation is rather awkward and time consuming to do by hand, but a spreadsheet can be set up to conveniently make the necessary calculations. The equations and calculation procedure are as follows:

The Redlich-Kwong compressibility factor, **Z**, is calculated as the maximum real root of the equation: $\mathbf{Z^3 - Z^2 - qZ - r = 0}$, where $\mathbf{r = A^2B}$ and $\mathbf{q = B^2 + B - A^2}$, with $\mathbf{A^2 = 0.42747 P_R/T_R^{2.5}}$ and $\mathbf{B = 0.08664\ P_R/T_R}$

To find the maximum real root, first the parameter C is calculated, where:
$\mathbf{C = (f/3)^3 + (g/2)^2}$, with $\mathbf{f = (-3q - 1)/3}$ and $\mathbf{g = (-27r - 9q - 2)/27}$

If $\mathbf{C > 0}$, then there is one real root,

$$\mathbf{Z = (-g/2 + C^{1/2})^{1/3} + (-g/2 - C^{1/2})^{1/3} + 1/3}$$

If $\mathbf{C < 0}$, then there are three real roots, given by:

$$\mathbf{Z_k = 2(-f/3)^{1/2}\cos[(\phi/3) + 2\pi(k - 1)/3] + 1/3}$$

With $\mathbf{k = 1, 2, 3}$ and $\mathbf{\phi = \cos^{-1}\{[(g^2/4)/((-f^3)/27)]^{1/2}\}}$

These equations & this method of calculating a value for the compressibility factor are described at:

The only input parameters needed to calculate the compressibility factor of a gas by the Redlich-Kwong method are the temperature and pressure at which the compressibility factor is to be calculated, along with the gas molecular weight, critical temperature and critical pressure.

However, there are 8 additional parameters introduced (**B, A², r, q, f, g, C,** and ϕ) and there are also quite a few steps in the solution, so it is not a trivial calculation to carry out by hand. An Excel spreadsheet is a very convenient tool to use in carrying out this calculation, as illustrated in **Example #3** and **Example #4**.

Example #3:

a) Calculate the density of air at -17.00 ºF & 20.00 psig, using the compressibility factor calculated by the Redlich-Kwong method described above.   Assume local atmospheric pressure is 14.7 psi.

b) Compare the results from part (a) with the density calculated assuming ideal gas behavior in **Example #1**.

Solution:

a)   The reduced temperature and reduced pressure can be calculated, using the gas temperature and pressure and the critical temperature and critical pressure of the gas, as follows:

$$\mathbf{T_R} = (-17.00 + 459.67)/(-220.9 + 459.67) =$$
$$\mathbf{1.854} \; \mathbf{P_R} = 34.7/547 = \mathbf{0.06344}$$

A spreadsheet screenshot is shown in **Figure 3** with the calculation of the compressibility factor and the density for air at -17 ºF and 20 psig. Note that the required user inputs (in the blue cells) are the gas molecular weight, temperature and pressure and the critical temperature and critical pressure of the gas. The inputs shown are for air at -17 ºF and 20 psig (34.7 psia).

The worksheet makes quite a few calculations in the yellow cells, starting with calculation of the reduced temperature ($\mathbf{T_R}$) and reduced pressure ($\mathbf{P_R}$). The calculated values of $\mathbf{T_R}$ and $\mathbf{P_R}$ are used to calculate the values of the 8 constants ($\mathbf{B, A^2, r, q, f, g, C}$, and $\boldsymbol{\phi}$).

If $\mathbf{C > 0}$, then the spreadsheet calculates the single real root of the equation, $\mathbf{Z^3 - Z^2 - qZ - r = 0}$, (which is the value of the compressibility factor, $\mathbf{Z}$) using the equation shown above ($\mathbf{Z = (-g/2 + C^{1/2})^{1/3} + (-g/2 - C^{1/2})^{1/3} + 1/3}$).

If **C < 0**, then the spreadsheet calculates the three real roots of the equation, using the equation given above, and the maximum of those three roots is the value for the compressibility factor, **Z**.

For **Example #3**, as shown in the screenshot, C > O and this results in a value of **0.997** for the compressibility factor, **Z**, and air density of **0.00659 slugs/ft³ = 0.21214 lbm/ft³**.

b) The value calculated for **Z** is very close to 1 and thus the calculated value of air density is quite close to that calculated with the Ideal Gas Equation in **Example #1** as shown below:

Ideal Gas Law: ρ = **0.00658 slugs/ft³ = 0.2116 lbm/ft³**

Redlich-Kwong: ρ = **0.00659 slugs/ft³ = 0.2121 lbm/ft³**

% difference = (0.2121 – 0.2116)/0.2121 = **0.24 %**

Calculation of the Density of a Gas Using its Compressibility
Factor and the Redlich-Kwong Equation of State - U.S. Units

Instructions: Enter input data for your calculation into the blue boxes. The
spreadsheet will carry out the calculations in the yellow boxes.

Inputs

			Calculations		
Gas Molec. Wt.. **MW** =	28.96		Absolute Gas Temp., **T** =	442.7	°R
Gas Temperature **t** =	-17	°F	Absolute Gas Temp., **T** =	245.9	K
Abs. Gas Pressure **P** =	34.7	psia	Abs. Gas Pressure **P** =	2.36	atm
Gas Critical Temp., **tc** =	-220.9	°F	Abs. Critical Temp., **Tc** =	132.7	K
Gas Critical Press., **Pc** =	547	psia	Abs. Critical Press., **Pc** =	37.21	atm
Ideal Gas Law Const., **R** =	345.23		Equation Constant **B** =	2.965E-03	
	psia-ft^3/slugmole-°R		Equation Constant **A^2** =	5.794E-03	
Calculations			Equation Constant **r** =	1.718E-05	
Reduced Temperature, **T$_R$** =	1.854		Equation Constant **q** =	-2.821E-03	
Reduced Pressure, **P$_R$** =	0.06344		Equation Constant **f** =	-3.305E-01	
P$_R$/(0.5T$_R$) =	0.07		Equation Constant **g** =	-7.315E-02	
			Calculated Value of **C** =	5.553E-07	

NOTE: For accurate calculations with the Redlich-
Kwong Eqn, it is recommended that P$_R$ be less than
half of T$_R$. The ratio shown in cell C19 should be
kept below 1 to get the best accuracy for the
calculated compressibility factor and gas density.

Equation Constant ϕ =		
Z$_1$ =		
Z$_2$ =		
Z$_3$ =		

Calculated Value of Compressibility Factor **Z** = 0.997

Gas density, ρ =   0.00659   slugs/ft^3 =   0.2121   lb$_m$/ft^3

Figure 3. Screenshot of Solution to **Example #3 (a)**

Note that for accurate calculations with the Redlich-Kwong equation, it is
recommended that **P$_R$** be less than half of **T$_R$**. In the **Figure 3** spreadsheet
screenshot the ratio **P$_R$/0.5T$_R$** is calculated to facilitate checking whether this
requirement has been met.

Example #4:

a) Calculate the compressibility factor and density of air at -129 ℃ and 20 bar, using the Redlich-Kwong method.

b) Compare the results from Part (a) with the density calculated assuming ideal gas behavior.

Solution:

a) The reduced temperature and reduced pressure can be calculated as follows:

$$T_R = (-129 + 273.15)/(-140.5 + 273.15) \ = \ \textbf{1.087}$$
$$P_R = (20 \text{ bar})(0.98682 \text{ atm/bar})/37.25 \text{ atm} \ = \ \textbf{0.5299}$$

A spreadsheet screenshot is shown in **Figure 4** with the calculation of the compressibility factor and the density for air at -129 ℃ and 20 bar pressure. The user inputs are shown in the cells at the upper left. Calculation of the reduced temperature and reduced pressure is shown below the user inputs and calculation of the various constants is shown in the cells at the right. The results, shown at the bottom of the screenshot are:

$$Z = \textbf{0.845} \quad \text{and air density} = \textbf{57.17 kg/m}^3$$

b) The value of **0.845** calculated for **Z** shows that the calculated value of air density will be somewhat different than that calculated with the Ideal Gas Equation. The results, shown at the bottom of the screenshot, are:

Ideal Gas Law: $\rho \ = \ \textbf{48.325 kg/m}^3$

Redlich-Kwong: $\rho \ = \ \textbf{57.170 kg/m}^3$

% difference $= \ (57.170 - 48.325)/57.170 \ = \ \textbf{15.5 \%}$

Calculation of the Density of a Gas Using its Compressibility Factor and the Redlich-Kwong Equation of State - S.I. Units

Instructions: Enter input data for your calculation into the blue boxes. The spreadsheet will carry out the calculations in the yellow boxes.

Inputs

			Calculations		
Gas Molec. Wt.. MW =	28.96		Absolute Gas Temp., T =	144.2	K
Gas Temperature t =	-129	°C	Abs. Gas Pressure P =	19.74	atm
Abs. Gas Pressure P =	20	bar	Abs. Gas Pressure P =	2000.0	kPa
Gas Critical Temp., tc =	-140.5	°C	Abs. Critical Temp., Tc =	132.7	K
Gas Critical Press., Pc =	37.25	atm (abs)	Equation Constant B =	4.225E-02	
Ideal Gas Law Const., R =	8.3145		Equation Constant A^2 =	1.840E-01	
	kN-m/kgmole-°K		Equation Constant r =	7.774E-03	

Calculations

			Equation Constant q =	-1.400E-01	
Reduced Temperature, T_R =	1.087		Equation Constant f =	-1.934E-01	
Reduced Pressure, P_R =	0.5299		Equation Constant g =	-3.519E-02	
$P_R/(0.5T_R)$ =	0.98		Calculated Value of C =	4.183E-05	

NOTE: For accurate calculations with the Redlich-Kwong Eqn, it is recommended that P_R be less than half of T_R. The ratio shown in cell C19 should be kept below 1 to get the best accuracy for the calculated compressibility factor and gas density.

Equation Constant ϕ =	
Z_1 =	
Z_2 =	
Z_3 =	

Calculated Value of Compressibility Factor Z = 0.845

Gas density, ρ = 57.170 kg/m³

Ideal Gas Density (for Z = 1). This value can be used if you don't know the critical temperature and critical pressure of the gas, but are confident that the gas exhibits ideal gas behavior

Ideal Gas density, ρ = 48.325 kg/m³

Figure 4. Screenshot of Solution to **Example #4 (a)**

6.   Calculation of Gas Viscosity by Sutherland's Formula

The Sutherland Formula provides a means for calculating the viscosity of a gas if the value of the Sutherland's Constant is known for that gas along with a value of the viscosity of that gas at some reference temperature. Sutherland's Formula is:

$$\mu = \mu_o \frac{T_o + C}{T + C} \left(\frac{T}{T_o}\right)^{3/2}$$

The parameters in this equation are:

T is the temperature of the gas, **K**

T$_o$ is a reference temperature, **K**

μ_o is the viscosity of the gas at **T$_o$** in any units

μ is the viscosity of the gas at **T** in the same units as μ_o

C is the Sutherland constant for the gas, **K**

Table 3 gives values of the Sutherland constant, the temperature range for that constant and the viscosity at three reference temperatures for 25 gases. These values can be used together with Sutherland's formula to calculate the viscosity of any of the gases in the table at any temperature within the specified temperature range. References #2, #3, and #4 are sources for the information in **Table 3**. Note that the reference temperature closest to the gas temperature (and the associated gas viscosity) should be used for the Sutherland's Formula calculation.

Table 3. Constants for Sutherland's Formula

Gas	Sutherland's Constant, K	Temperature Range, °C	Gas Viscosity, μ_o, cP		
			t_o = 20 °C	t_o = 50 °C	t_o = 100 °C
Acetylene	320	20 - 100	0.0100	0.01111	0.0127
Air	113	20 - 100	0.0183	0.0198	0.0220
Ammonia	503	25.1 - 300	0.0099	0.0110	0.0129
Argon	148	20 - 100	0.0223	0.0242	0.0271
Benzene	300	20 - 100	0.0075	0.0081	0.0094
Butane	270	20 - 100	0.00742	0.00814	0.00932
Carbon dioxide	253	20 - 100	0.0147	0.0161	0.0184
Carbon monoxide	102	20 - 100	0.0174	0.0188	0.0209
Chlorine	345	20 - 100	0.0132	0.0145	0.0167
Ethane	252	20 - 250	0.00921	0.0101	0.0115
Ethylene	225	20 - 250	0.0100	0.0110	0.0126
Helium	72.9	20 - 100	0.0196	0.0210	0.0232
Hydrogen	66.8	20 - 100	0.0088	0.0094	0.0104
Hydrogen Chloride	362	21 - 250	0.0144	0.0159	0.0183
Hydrogen Sulphide	331	17 - 100	0.0124	0.0137	0.0159
Methane	169	20 - 100	0.0110	0.0120	0.0135
Nitric Oxide	133	20 - 100	0.0175	0.0188	0.0210
Nitrogen	104.7	19.8 - 825	0.0189	0.0204	0.0228
Nitrous Oxide	263	20 - 100	0.0147	0.0161	0.0184
Oxygen	125	14.8 - 829	0.0203	0.0220	0.0246
Propane	150	20 - 100	0.00821	0.00896	0.0102
Propylene	250	15.3 - 100.1	0.00847	0.00934	0.0107
Steam	260	20 - 100	0.0097	0.0106	0.0124
Sulphur Dioxide	404	20 - 100	0.0127	0.0140	50
Xenon	252	15.3 - 100.1	0.0228	0.0251	0.0288

Example #5:   Calculate the viscosity of methane at 110 °F using Sutherland's formula and values from **Table 3**.

Solution: Converting 110 °F to °C gives: (110 – 32)/1.8 = 43.33 °C. Thus the reference temperature closest to the gas temperature is 50 °C, so from

Table 3, we will use t_o = 50 ºC and μ_o = 0.0120 cP.   Also from **Table 3**, the Sutherland's Constant for methane is 169.   The gas temperature and reference temperature must be in K (degrees Kelvin) for use in Sutherland's Formula to calculate the gas viscosity. The temperature conversions can be made as follows:

Gas temperature = (110 + 459.67)/1.8 = 316.5 K
Reference temperature = (50 + 273.15) = 323.15 K

Now, substituting into Sutherland's Formula to calculate the viscosity of methane at 110º F gives:

$\mu = 0.012[(323.15 + 169)/(316.5 + 169)][(316.5/323.15)^{1.5}]$ = **0.0118 cP**

Converting to typical U.S. units of lb-s/ft^2 gives:

$\mu = (0.0118)(2.08854 \times 10^{-5})$ = **2.46 x 10^{-7} lb-s/ft^2**

Calculation of a gas viscosity using Sutherland's Formula can conveniently be done using an Excel spreadsheet. **Figure 5** shows a screenshot of a spreadsheet solution to **Example #5**.

Note that, in the screenshot, the given information is entered into the blue cells on the left side of the screenshot and the spreadsheet makes the calculations in the yellow cells to make the necessary temperature conversions, calculate the gas viscosity and convert to typical U.S. units.

Calculations in S.I. units would be very similar, but the unit conversions used for **Example #5** wouldn't be needed, because the values in Table 3 have units of ºC, K, and cP. This type of calculation is illustrated in **Example #6**.

Figure 5. Screenshot of Solution to **Example #5**

Example #6: Calculate the viscosity of methane at 60º C using Sutherland's formula and values from **Table 3**.

Solution: The reference temperature closest to the gas temperature is 50 ºC, so from **Table 3**, we will use $t_o = 50$ ºC and $\mu_o = 0.0120$ cP. Also from Table 3, the Sutherland's Constant for methane is 169. The gas temperature and reference temperature must be in K (degrees Kelvin) for use in Sutherland's Formula to calculate the gas viscosity. The temperature conversions can be made as follows:

Gas temperature $= (60 + 273.15) = 333.15$ K
Reference temperature $= (50 + 273.15) = 323.15$ K

Now, substituting into Sutherland's Formula to calculate the viscosity of methane at 60º C gives:

$\mu = 0.012[(323.15 + 169)/(333.15 + 169)][(333.15/323.15)^{1.5}] =$ **0.0123 cP**

7.   Calculation of Air Viscosity at Given Temperature and Pressure

With the following equations, it is possible to calculate the viscosity of air for specified values of the air temperature and pressure. The source for these equations is **Reference #6** at the end of this course. Note that η_o and $\Delta\eta$ are simply functions of reduced temperature and reduced density that are used in calculating air viscosity. They do not have any physical interpretation.

air viscosity $= (1.2867/10^7)[\eta_o(T_r) + \Delta\eta(\rho_r)]$ lb-sec/ft^2

$\eta_o(T_r) = 0.128517T_r + 2.60661T_r^{0.5} - 1.0 - 0.709661T_r^{-1}$

$\qquad\qquad\qquad + 0.662534T_r^{-2} - 0.197846T_r^{-3} + 0.00770147\ T_r^{-4}$

$\Delta\eta(\rho_r) = 0.465601\rho_r + t\ 1.26469\rho_r^2 - 0.511425\rho_r^3 + 0.274600\rho_r^4$
$T_r = T/238.5$ (T in ºR) $\rho_r = \rho/0.6096$ (ρ in slugs/ft^3)

Note that ρ is the density of the air at the specified temperature and pressure, which can typically be calculated as described in Section 4 for ideal gas behavior. If the air cannot be treated as an ideal gas for the specified temperature and pressure, then the density would need to be calculated as described in Section 5 above.

Example #7:   Calculate the viscosity of air at 50.0ºF and 40.0 psig at a location where atmospheric pressure is 14.7 psi.

Solution: The absolute air temperature can be calculated as:
$$\mathbf{T} = 50.0 + 459.7 = \mathbf{509.7\ ^oR}$$

The absolute air pressure is:
$$\mathbf{P} = 40.0 + 14.7 = \mathbf{54.7\ psia}$$

The air density can then be calculated as: $\rho = P(MW)/RT$

$$\rho = (54.7)(28.97)/[(345.23)(509.7) = \textbf{0.0090 slugs/ft}^3$$

T_r and ρ_r can now be calculated as follows:

$$T_r = T/238.5 = 509.7/38.5 = \textbf{2.137}$$

$$\rho_r = \rho/0.6096 = 0.0090/0.6096 = \textbf{0.00715}$$

Now $\eta_o(T_r)$ and $\Delta\eta(\rho_r)$ can be calculated with the equations shown above to give:

$$\eta_o(T_r) = \textbf{2.878} \qquad \text{and} \qquad \Delta\eta(\rho_r) = \textbf{0.00715}$$

Finally, the air viscosity can now be calculated using the first equation shown in this chapter, to give:

air viscosity $= (1.2867/10^7)\textbf{[}\boldsymbol{\eta_o}\textbf{(T}_r\textbf{)} + \boldsymbol{\Delta\eta(\rho_r)}\textbf{]}$

$$= (1.2867/10^7)(2.878 + 0.00715) = \textbf{3.71 x 10}^{\textbf{-7}} \text{ lb-sec/ft}^2$$

As you may expect, this set of calculations also can be conveniently done with a properly set up spreadsheet. **Figure 6** shows a screenshot of a spreadsheet solution to **Example #6**.

Only three user inputs are needed. They are the temperature of the air, the gage pressure of the air, and atmospheric pressure. These three inputs are entered in the three blue cells in the upper left part of the screenshot in **Figure 6**. The spreadsheet then calculates the absolute air temperature and pressure, the air density, T_r, ρ_r, $\eta_o(T_r)$, $\Delta\eta(\rho_r)$, and finally the air viscosity at the specified temperature and pressure. Note that the calculated value for the air viscosity at 50°F and 40 psig is shown as 3.17 x 10^-7 lb-sec/ft², the same as shown in the calculations above.
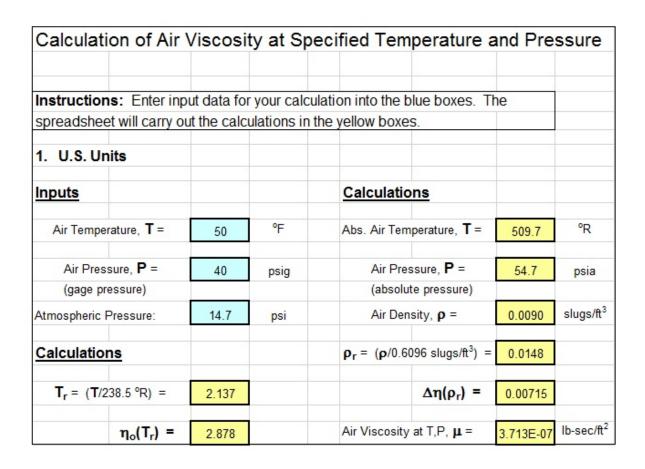
Calculation of Air Viscosity at Specified Temperature and Pressure

Instructions: Enter input data for your calculation into the blue boxes. The spreadsheet will carry out the calculations in the yellow boxes.

1. U.S. Units

Inputs

Inputs			Calculations		
Air Temperature, **T** =	50	°F	Abs. Air Temperature, **T** =	509.7	°R
Air Pressure, **P** = (gage pressure)	40	psig	Air Pressure, **P** = (absolute pressure)	54.7	psia
Atmospheric Pressure:	14.7	psi	Air Density, ρ =	0.0090	slugs/ft^3
Calculations			ρ_r = (ρ/0.6096 slugs/ft^3) =	0.0148	
T_r = (T/238.5 °R) =	2.137		$\Delta\eta(\rho_r)$ =	0.00715	
$\eta_o(T_r)$ =	2.878		Air Viscosity at T,P, μ =	3.713E-07	lb-sec/ft^2

Figure 6. Screenshot of Solution to **Example #7**

For calculations in S.I. units some of the constants in the equations given above are changed. Specifically, the equations for T_r, ρ_r, and the air viscosity change, while the equations for $\eta_o(T_r)$ and $\Delta\eta(\rho_r)$ remain the same. The S.I. equations with changed constants are:

$$T_r = T/132.5 \quad (T \text{ in K}) \qquad\qquad \rho_r = \rho/314.3 \quad (\rho \text{ in kg/m}^3)$$

And

$$\text{air viscosity} = (6.16090/10^6)[\eta_o(T_r) + \Delta\eta(\rho_r)] \quad \text{Pa-s}$$

Example #8:   Calculate the viscosity of air at 327 °C and 20 bar gage pressure at a location where atmospheric pressure is 101.325 kPa.

Solution: The absolute air temperature can be calculated as:
$$T = 327 + 273.15 = \textbf{600.2 K}$$

The absolute air pressure is:
$$P = (20)(100) + 101.325 = \textbf{2101.3 kPa}$$

The air density can then be calculated as: $\rho = \textbf{P(MW)/RT}$

$$\rho = (2101.3)(28.97)/[(8.3245)(600.2) = \textbf{12.20 kg/m}^3$$

T_r and ρ_r can now be calculated as follows:

$$T_r = T/132.5 = 600.2/132.5 = \textbf{4.529}$$

$$\rho_r = \rho/314.3 = 12.20/314.3 = \textbf{0.0388}$$

Now $\eta_o(T_r)$ and $\Delta\eta(\rho_r)$ can be calculated with the equations shown above to give:
$$\eta_o(T_r) = \textbf{5.003} \quad \text{and} \quad \Delta\eta(\rho_r) = \textbf{0.01995}$$

Finally, the air viscosity can now be calculated using the air viscosity equation shown above, to give:

air viscosity $= (6.16090/10^6)\textbf{[}\eta_o\textbf{(}T_r\textbf{)} + \Delta\eta(\rho_r)\textbf{]}$

$$= (6.16090/10^6)(5.003 + 0.01995) = \textbf{3.095 x 10}^{-5} \text{ Pa-s}$$

Figure 7 shows a screenshot of a spreadsheet with the solution to **Example #8.**

Calculation of Air Viscosity at Specified Temperature and Pressure

Instructions: Enter input data for your calculation into the blue boxes. The spreadsheet will carry out the calculations in the yellow boxes.

2. S.I. Units

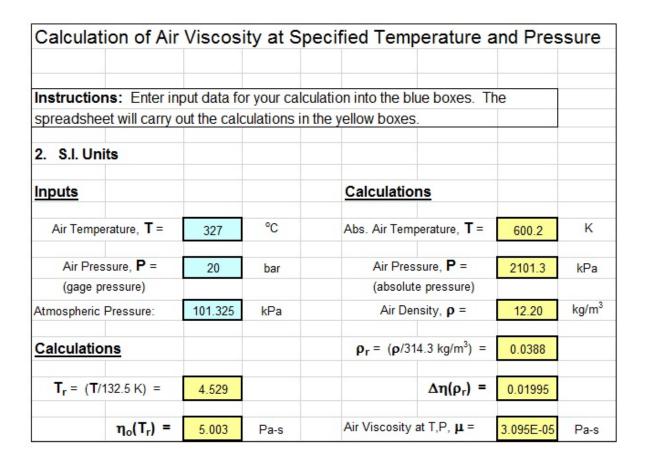| Inputs			Calculations		
Air Temperature, T =	327	°C	Abs. Air Temperature, T =	600.2	K
Air Pressure, P =	20	bar	Air Pressure, P =	2101.3	kPa
(gage pressure)			(absolute pressure)		
Atmospheric Pressure:	101.325	kPa	Air Density, ρ =	12.20	kg/m^3
Calculations			$\rho_r = (\rho/314.3 \text{ kg/m}^3)$ =	0.0388	
$T_r = (T/132.5 \text{ K})$ =	4.529		$\Delta\eta(\rho_r)$ =	0.01995	
$\eta_o(T_r)$ =	5.003	Pa-s	Air Viscosity at T,P, μ =	3.095E-05	Pa-s

Figure 7. Screenshot of Solution to **Example #8**

8.   Summary

The density and viscosity of a gas at specified temperature and pressure can be calculated using the methods covered in this course. Gas density can be calculated using the Ideal Gas Law if the gas temperature is sufficiently greater than the critical temperature and the gas pressure is sufficiently less than the critical pressure. If the gas temperature and/or pressure are such that the Ideal Gas Law cannot be used then the gas density can be calculated using the calculated compressibility factor. In this course, use of the Redlich-Kwong equation of state to calculate the compressibility factor was presented and illustrated with example calculations.

The viscosity of a gas at specified gas temperature can be calculated using Sutherland's Formula as presented and illustrated with examples. Equations were also presented for calculation of the viscosity of air at specified temperature and pressure.

Use of both U.S. units and S.I. units was presented and illustrated with examples for all of the calculations.

9.   References

1.   Redlich-Kwong Equation of State Calculations, www.polymath-software.com/ASEE2007/PDF1.pdf

2. Chapman, S. & Cowling, T.G., The Mathematical Theory of Non-Uniform Gases, 3rd Ed., 1970, Cambridge Mathematical Library

3. National Physical Laboratory, Kaye & Laby, Tables of Physical and Chemical Constants, Chapter 2.2, Subsection 2.2.3, http://www.kayelaby.npl.co.uk/general_pysics/2_2/2_2_3.html

4. Engineering Toolbox website: www.engineeringtoolbox.com/gases-absolute-dynamic-viscosity-d_1888.html.

5. Green, Don W. and Perry Robert H., Perry's Chemical Engineers' Handbook, 8th Ed, Table2-312, McGraw-Hill

6. Kadoya, K, Matsunaga, N, and Nagashima, A, *Viscosity and Thermal Conductivity of Dry Air in the Gaseous Phase*, J. Phys. Chem. Ref. Data, Vol 14, No. 4, 1985. https://srd.nist.gov/JPCRD/jpcrd283.pdf

7. Bengtson, Harlan H., Sutherland Formula Viscosity Calculator, an online blog article at www.EngineeringExcelTemplates.com.

8. Bengtson, Harlan H. Gas Compressibility Factor Calculator Excel Spreadsheet, an online blog article at www.EngineeringExcelSpreadsheets.com

9. Bengtson, Harlan H., Gas Property Calculator Spreadsheets, An Amazon Kindle E-Book.